Preference-Based Optimisation in Group Decision-Making

Addendum to Chapter 6 of

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Summary

This Addendum extends, broadens, and optimises IMAP/ODESYS 1.0, originally presented in Chapter 6 of Open Design Systems (Wolfert, 2023). It enhances its applicability and repositions ODESYS within the landscape of contemporary research and practice. A concise summary is provided, followed by the introduction of the ODESYS structure-in-FIVES formulation: a novel Open Design & Decision Systems framework.

Conventional multi-objective optimisation approaches (e.g., MOO-CP or MIP) typically fail in group decision-making by aggregating heterogeneous objectives without a valid preference foundation, producing Pareto-dominance sets instead of a unique, actionable decision. Because only humans can define objectives, their preferences constitute the only legitimate basis for decision-making. Accordingly, four essential conditions for complex, multi-level design–decision systems are established: (1) **Preference-Key** — all objectives, constraints, and trade-offs are evaluated within a unified preference domain using valid preference function modelling (PFM); (2) **Integration** — feasible system performance ("object-capability") and acceptable actor preferences ("subject-desirability") coexist within a single design–decision space; (3) **Association** — actors can freely specify their individual preferences and weights, enabling consistent aggregation towards group-optimal decision-making; and (4) **Uniqueness** — the solver identifies a single best-fit solution with maximum aggregated preference.

The open design and decision system (ODESYS) methodology, employing the **IMAP** (Integrative Maximisation of Aggregated Preferences) solver that satisfies these conditions, enables fully integrated multi-objective design optimisation and multi-criteria decision-making (MODO-MCDM). Its extension within the **ODESYS/FIVES** formulation, introduced here, broadens applicability while excelling in elegant simplicity. It explicitly operationalises affine preference aggregation and preserves equivalence with previously validated ODESYS 1.0 results. By mapping all system behaviour into a unified, goal-oriented preference-performance domain and directly aggregating actor preferences via PFM, ODESYS/FIVES/IMAP confronts system complexity and delivers a single, best-fit solution—even for highly constrained combinatorial group decision problems. In doing so, it guarantees feasible and acceptable outcomes, providing pure group design–decision support that surpasses conventional methods, which may generate numerical results yet lack group decision validity.

Two illustrative applications—a marine installation and a multi-constrained vessel allocation problem—demonstrate how ODESYS/FIVES produces a unique, preference-based solution that integrates stakeholder interests across system levels. This approach effectively transforms multi-objective optimisation into pure group decision-making, achieving a best-fit-for-common-purpose within socio-physical reach.

Introduction

Across systems engineering, design, and decision science, multi-objective optimisation (MOO) is widely recognised as a core group decision-support activity for confronting complexity. Contemporary problems involve heterogeneous objectives—e.g., availability, affordability, sustainability, and aesthetic value—evaluated by multiple stakeholders under uncertainty and dynamic conditions. Optimisation must therefore move beyond purely physical performance metrics and explicitly incorporate stakeholder preferences (utility or value), as only these provide a valid basis for goal-oriented choice. Separation between optimisation and preference evaluation undermines the design–decision outcome, since only the human-subject is goal-oriented and can set objectives, not the physical-object.

Foundational work, such as Decision-Based Design (DBD) (Hazelrigg, 1998), Value Engineering (King, 2000), and utility-based design formulations (Thurston, 2011), highlighted the necessity of preference-aware MCDM. However, these approaches fail to embed preferences as a rigorous basis for group decision-making. Preference measurement was formalised through Barzilai's Preference Function Modelling (PFM) theory (Barzilai, 2010, 2022), demonstrating that preferences reside in a one-dimensional affine space and admissible transformations are strictly affine. Despite this, many aggregation practices remain mathematically invalid or decision-ambiguous. Recently, (Wolfert, 2026) demonstrated mathematically valid aggregation of preferences in MCDM using PFM.

Classical systems engineering design optimisation texts (Blanchard & Fabrycky, 2021; Martins & Ning, 2022) and management science references (Hillier & Lieberman, 2021) provide algorithmic multi-objective solutions (scalarisation, ε-constraint, Pareto heuristics), but they focus on numerical optima rather than preference-based MCDM. Consequently, solutions often yield Pareto sets or single optima lacking contextual preference meaning. Contemporary literature (Ferdous et al., 2024; Pajasmaa et al., 2025) highlights the need for a framework that properly integrates preference, acceptability, performance, and feasibility to identify a design–decision vector that maximises aggregated preference.

In practice, MOO for multi-constrained problems uses LP/MIP, hybrid MOO-MCDM frameworks, and constraint programming (CP) (Hillier & Lieberman, 2021; Martins & Ning, 2022), often realised via weighted additive rankings, ε-constraints, lexicographic ordering, Pareto sets, or hierarchical pruning. These methods explore performance spaces but do not deliver a unique, group decision-valid solution; they optimise measures, not human, goal-oriented preferences. Moreover, exploring the full feasible design space is inefficient since only the subset of designs aligned with stakeholder preferences is relevant for effective decision-making.

Similarly, advances in parametric and computational design (Block, 2013; Woodbury, 2010) enable systematic exploration of large design spaces through structured parameters and constraint-based modelling. While effective for evaluating technical and performance criteria (structural performance, sustainability, constructability, etc.), these methods remain fundamentally performance-driven, do not internalise stakeholder desirability or acceptability, and cannot produce a unique, decision-valid best-fit solution. They generate candidate designs requiring external interpretation, highlighting the gap between exploration and actual decision-making.

To substantiate these findings, the following sections overview contemporary MOO methods for group decision-making, highlight gaps in research and practice, and present a development statement, including four conditions for preference-based optimisation that enable pure group decision-making.

Contemporary MOO for group decision-making overview

Persistent separation of preferences and system performance remains a key limitation of classical optimisation. Preferences are often applied after technical feasibility is established, rather than embedded within a unified design–decision space, as in systems engineering and performance-driven design frameworks (Blanchard & Fabrycky, 2021; Weck et al., 2011). This persists in Bayesian optimisation (Ahmadianshalchi et al., 2023), generative design systems (Chen & Xu, 2025), and dynamic preference models (Arezoomand, 2021; Kim et al., 2014; Regenwetter et al., 2022; Saadi et al., 2024), as well as in interactive evolutionary algorithms (Branke et al., 2016; Thiele et al., 2009) and Choquet-integral or constraint-reformulation approaches (Castellanos-Alvarez et al., 2021; Hou et al., 2020). When feasible performance ("what it can") and acceptable stakeholder preferences ("what is wanted") are not integrated, changes in feasibility, acceptability, or project state fail to propagate coherently through the decision-making process. Moreover, exploring the full feasible design space is inefficient, since only the subset of designs aligned with stakeholder preferences is truly relevant, leading to fragmented solution spaces with no convergent best-fit outcome.

Stakeholders and actor dynamics have become central in post-2010 research, reflecting the socio-technical nature of design decisions and the importance of stakeholder involvement (Arkesteijn et al., 2017; Yang et al., 2022; Zhilyaev et al., 2022). Participative, negotiation-based, and group decision frameworks improve descriptive realism (Chou et al., 2021; Du & Jiao, 2022; Lagaros et al., 2023; Pérez et al., 2023; Qiao et al., 2024; Rahimi et al., 2022; Shavazipour, 2025), yet preferences are typically episodic inputs rather than dynamic variables integrated throughout optimisation. Optimisation and decision-making therefore remain conceptually and mathematically segregated (Adekoya & Helbig, 2023; Chen & Xu, 2025). While contemporary works (Wang et al., 2025; Zhilyaev et al., 2022) provide initial integration steps, a fully unified incorporation of engineering performance, feasibility, stakeholder preferences, and acceptability within a single associative preference-based design–decision space remains largely absent from mainstream MOO practice. Consequently, the a priori identification of a best-fit-for-common-purpose solution remains untapped, stakeholder needs insufficiently integrated, and the notion of a free, best-for-project choice vulnerable to becoming curated and illusory.

The absence of a unified preference domain for heterogeneous objectives remains a core limitation in multi-objective optimisation (MOO). Methods either resort to hierarchical or semi-multi-objective formulations, or rely on monetisation as a workaround for commensurability, despite longstanding critiques from decision theory, cost–benefit-focused design, and socio-technical research (Du & Jiao, 2022; Hirsch Hadorn, 2022). Even recent hybrid MOO–MCDM frameworks still layer a posteriori MCDM steps to reconcile fundamentally incomparable objectives (Chen & Xu, 2025; Ferdous et al., 2024). Although explicit preference modelling is increasingly recognised as essential, systematic use of continuous, individually weighted preference functions grounded in rigorous measurement theory—and applicable across economic, technical, environmental, and social dimensions—remains rare. Notable recent efforts (Wang et al., 2025) integrate surrogate-assisted search with preference cues for expensive MOO problems while avoiding ideal point estimation, yet they remain objective-anchored and set-oriented rather than providing a fully associative, decision-valid preference domain (Arezoomand, 2021; Lee et al., 2011; Messac, 1996; Saadi et al., 2024; Wang et al., 2025). In practice, heterogeneous objectives are treated in isolated spaces, with commensurability enforced via scaling, weighting, or dominance filtering rather than a unified preference domain. This confirms that the lack of a common preference domain is a persistent, cross-methodological challenge rooted in fundamental decision-theoretic limits, not mere algorithmic detail (Gunantara, 2018).

Unique preference aggregation is central to MOO–MCDM, as its mathematical validity determines the meaningfulness of decision outcomes. Nevertheless, most classical and post-2010 MOO–MCDM frameworks rely on normalisation, weighted sums, surrogate scores, or composite indices (Adekoya & Helbig, 2023; Dehshiri et al., 2022; Ferdous et al., 2024; Kaddani et al., 2017; Zeng et al., 2025), implicitly assuming ratio- or interval-scale properties that preference data do not possess. Preference, however, is not a physical property but a subjective construct of the mind, expressing an individual's free ordering of alternatives within a given context and defining a decision space that is inherently relational, individual, and situation-dependent. More expressive methods—such as discrete choice models, network-based decision frameworks, and probabilistic preference embeddings (Sha et al., 2023)—increase representational or situational richness without resolving this foundational measurement inconsistency. Likewise, weighted aggregation with partial preference information, fuzzy–stochastic methods, and Pareto-front transformations continue to rely on normalisation, fuzzy membership functions, or transformed dominance relations—approaches that are operationally convenient yet methodologically fragile. Consequently, mathematically admissible and preference-consistent aggregation remains rare. Recent analyses (Pajasmaa et al., 2025) confirm that none of these approaches simultaneously satisfy the axioms of rigorous preference function modelling (PFM), which establishes that only differences between preference values are meaningful, admissible transformations are affine, and aggregation must preserve zero-reference stability and commensurability (Barzilai, 2010, 2022; Wolfert, 2026).

Pareto remains the dominant epistemic anchor in multi-objective optimisation (Marler & Arora, 2004), including contemporary preference-aware variants (Ahmadianshalchi et al., 2023; Pajasmaa et al., 2025; Zhao et al., 2025). Most methods terminate in Pareto sets requiring a posteriori selection, negotiation, or heuristic filtering. As a dominance concept rather than a decision principle, Pareto optimality identifies non-inferior alternatives but cannot produce a unique, actionable solution—yet real-world design and governance demand a single best-fit-for-common-purpose outcome. Even enhancements—such as interactive evolutionary algorithms (Thiele et al., 2009), heuristic Pareto-front management (Kesireddy & Medrano, 2024), and hybrid aggregation-transform techniques (Zeng et al., 2025)—still yield sets or ranked subsets. Reviews (Gunantara, 2018) confirm Pareto-dominance-based sets dominate MOO, with preferences typically applied post hoc via weights, constraints, or surrogate indices. Pareto fronts are often mathematically flawed and produce infinite, supposedly equal alternatives, inconsistent with stakeholder-interpreted, single optimal solutions (Bai et al., 2015; Golany et al., 2006; Saad et al., 2018). Hybrid methods remain a posteriori, reinforcing the persistent absence of a unified preference domain (Gunantara, 2018).

Emerging preference-based optimisation research challenges traditional dominance-based constructs, including ideal and nadir points (Zhao et al., 2025), signalling a shift toward more explicitly preference-aware optimisation paradigms. Advanced approaches—such as interactive evolutionary algorithms (Branke et al., 2016), constraint-enhanced preference integration (Hou et al., 2020), and expensive preference-guided MOO methods (PMEGO) that avoid ideal points (Wang et al., 2025)—better incorporate stakeholder objectives and capture complex criterion interactions. However, even these state-of-the-art methods do not satisfy the axioms of mathematically valid preference aggregation (Pajasmaa et al., 2025; Wolfert, 2026). They remain fundamentally Pareto-set or interaction-based: preferences steer, restrict, or filter candidate solutions, but are not structurally inseparable from system behaviour and do not produce a unique, decision-theoretically grounded outcome. Despite growing recognition of preference awareness, pure preference-based optimisation—mathematically valid, preference-meaningful, non-monetary, non-Pareto-based, and capable of yielding a single best-fit-for-common-purpose solution—remains absent from mainstream design and project management practice.

To date, and to the best of the author's knowledge, the IMAP Open Design Systems–driven method is the only approach that operationalises such integrative and associative preference-based optimisation (van Heukelum et al., 2024; Wolfert, 2023), and this paper further extends IMAP to an even broader range of group design-decision applications, confronting systems complexity.

Research & Practice Gap

Across research and practice, persistent limitations remain in multi-objective design optimisation for group decision-making. Conceptually, preference-based design is widely acknowledged as necessary, yet current methods are incomplete. Pareto-dominance approaches structure search but cannot yield unique, decision-valid outcomes; preferences often guide exploration without full integration; aggregation methods frequently violate measurement-theoretic principles; and decision-making is deferred to episodic heuristics rather than being continuously embedded. Frameworks such as ODESYS introduced a unified preference-over-performance domain, formally integrating individual and acceptable stakeholder preferences with feasible system performance. Nevertheless, it remains partially objective-anchored, and its application to highly constrained socio-physical systems is under-explored. No framework yet fully realises a dynamically adaptive, mathematically rigorous preference-driven design–decision space capable of integrating system capability, feasibility, stakeholder desirability, and acceptability into a single best-fit-for-common-purpose solution.
In parallel, industry and public-sector practice faces a similar challenge. Existing methodologies remain fragmented, privileging either technical potential ('what can') or stakeholder desire ('what is wanted') without resolving their integration. Object-driven optimisation models technical performance but cannot generate meaningful best-fit-for-common-purpose outcomes; subject-driven methods—such as a posteriori ranking of pre-generated alternatives—simulate choice yet cannot guarantee actionable, feasible outcomes. Stakeholders are often confined to curated options, limiting decision freedom and potentially compromising results. Practitioners therefore require methods that embed capability and desirability within a shared, associative decision space, enabling transparent, substantiated decision-making while confronting complex socio-technical constraints. This preserves individual design freedom alongside collective alignment and legitimacy, ensuring the best-fit can be identified a priori.

Taken together, both research and practice gaps highlight the absence of frameworks capable of concurrent, associative, and fully integrative decision-making, in which stakeholder preferences, system feasibility, and performance converge to produce a unique, best-fit-for-common-purpose solution.

Development Statement

The limitations identified in both research and practice motivate the further evolution of the IMAP (Integrative Maximisation of Aggregated Preferences) solver and the early ODESYS framework (van Heukelum et al., 2024; Wolfert, 2023), which integrate acceptable stakeholder preferences and feasible system performance within a unified preference domain capable of producing a PFM-consistent best-fit design point. While ODESYS/IMAP 1.0 already delivered robust results, these formulations remained partially anchored in conventional multi-objective thinking.

The full potential of a **pure performance–preference approach**—eliminating the objective layer and enabling fully associative, group decision-valid MCDM for highly constrained system contexts—is realised here through an extended ODESYS operator. This formulation both preserves previously validated design–decision outcomes and broadens applicability across complex socio-physical systems. Formally, this development can be expressed as follows:

"There is a need for an open, integrative design–decision methodology that enables complex systems development across all relevant system levels, grounded in mathematically valid preference-based optimisation over system performance dimensions, and supporting transparent, associative group decision-making."

To realise this rigorously, a MOO method yields a unique, best-fit-for-common-purpose MCDM outcome if and only if it satisfies the following **four formal conditions**, which together define group decision-valid preference-based optimisation.

Condition 1 — Preference-Key

A multi-objective optimisation method is decision-valid only if all performance objectives, constraints, and trade-offs are represented and evaluated within a single, unified preference domain using a mathematically valid preference function model. Methods that rely on objective-space dominance relations (e.g., Pareto fronts), surrogate metrics, or invalid preference aggregation (e.g., direct weighted additive aggregation of heterogeneous objectives) therefore cannot yield a unique decision corresponding to the maximisation of aggregated preference.

Condition 2 — Integration

A decision-valid optimisation method integrates subject-preference with object-performance. Feasible system performance (capability: "what it can") and acceptable stakeholder preferences (desirability: "what they want") must reside within a single integrative design-decision solution space. The immediately given physical reality—representing the system's degrees of freedom ('infinite' possibilities)—must be coherently unified with human free will, expressed through individual actors' goal-oriented preferences (common purpose); only under this integration can multi-objective optimisation produce a consistent design–decision synthesis.

Condition 3 — Association
A group-decision-valid optimisation framework is genuinely participative only when each individual stakeholder decision-maker is free to specify their own preference representation, including relative weighting and acceptable bounds within the feasible performance space. This formulation enables coherent cooperation and active participation across multiple actors and performance dimensions, defining a best-for-group decision as the maximisation of aggregated individual preferences over that space. Only within such an associative framework can a group-optimal decision emerge: when each actor advances their legitimate interests yet is willing to concede on pure self-interest, the group outcome achieves a higher aggregated preference than any purely individualistic design-decision alternative.

Condition 4 — Uniqueness
A decision-valid optimisation method shall yield a consistent, unique best-fit-for-common-purpose solution that is invariant under admissible transformations of preferences, ensuring that the multi-objective optimisation results remain preference-meaningful and, therefore, group decision-support valid. Optimisation methods that return sets of non-equivalent solutions are merely numerical constructs that fail to support group decision-making.

Taking these as a starting point, and building on recent ODESYS/IMAP 1.0 developments, the novel ODESYS structure in FIVES elevates preference-guided optimisation into a fully associative, decision-valid method for complex systems development. It rigorously satisfies the four formal MOO group decision-validity conditions while preserving previously validated ODESYS outcomes. Moreover, it enables the resolution of highly constrained socio-physical design–decision problems, moving beyond the forced compromises inherent in conventional multi-objective, constraint-, and Pareto-based optimisation approaches toward a genuinely best-fit-for-common-purpose solution. By internalizing human preference, desirability, and acceptability directly within the feasible system performance space, ODESYS transforms parametric design exploration into a group decision-valid synthesis process embedded in reflective practice.

1. Open design and decision system

Let a complex, multifaceted problem, comprising multiple objects and subjects, be formulated as an open design and decision system (ODESYS) synthesis operator:

$$\mathrm{OD}\Big(P_{k,i}(\mathbf{F}(\mathbf{x},\mathbf{y},t));\ w'_{k,i}\Big), \qquad (\mathbf{x},\mathbf{y}) \in S_{f,a}. \tag{1}$$

Here, $\mathbf{F} = F_i(\mathbf{x},\mathbf{y},t)$, with $i = 1,\dots,I$ (where I is the total number of performance functions), denotes the system performance functions that describe the socio-physical functional behavior of the system — i.e., the system capabilities. These functions represent what the system *can* perform within the design and decision systems solution space $S_{f,a}$.
The vector $\mathbf{x}$ represents the 'controllable' design-decision vector, comprising both product and process variables that can be deliberately selected and adjusted by the decision maker. The vector $\mathbf{y}$ is the 'uncontrollable' parameter vector, containing contextual variables, including external product and process conditions that influence system behaviour but cannot be controlled. The variable t represents the system state in time. Depending on the nature

of the design problem, time may be explicitly modeled, yielding a time-dependent performance $\mathbf{F}(\mathbf{x}, \mathbf{y}, t)$; it may be fixed to a decision horizon $t = T^*$; or it may be absent, in which case the formulation reduces to $\mathbf{F}(\mathbf{x}, \mathbf{y})$.

In contrast to traditional optimisation approaches, these performance functions over the design-decision vector do not represent objectives by themselves. Goal-oriented decision-making is only enabled by transforming the performance functions into preference functions, given by the actor and allowing stakeholder-specific trade-offs to be evaluated within the ODESYS framework.

The preference function $P_{k,i}(\cdot)$ reflects the preference of a decision-making stakeholder $k = 1, \ldots, K$ (K = max. number of actors) with respect to performance function F_i. A preference expresses the relative desirability, *value*, or *utility* of a design alternative or decision option with respect to a functional performance, given its global participation and its local (weight) importance $w'_{k,i}$ across performance functions, where $\sum_i w'_{k,i} = 1$. When the stakeholder has no interest in a certain performance function, its weight is zero. The preferences function are determined by the individual actors themselves, allowing the preference function to be linear, non-linear, convex, concave, or piecewise.

Preference is not a physical property, but a subjective construct representing an individual's choices, thereby defining the decision space. The preference function $P_{k,i}$ maps an actor-independent performance value $f_i(\mathbf{x}, \mathbf{y}, t)$ to a corresponding preference score, reflecting the goal-oriented evaluation of actor k:

$$P_{k,i}\big(f_i(\mathbf{x}, \mathbf{y}, t)\big) \in [0, 100].$$

The relatively worst performance (minimum, least preferred) and best performance (maximum, most preferred) are assigned scores of 0 and 100, respectively (an arbitrary preference reference choice, since preferences have only relative meaning as part of a one-dimensional affine space (Wolfert, 2026)).

The ODESYS system is constrained by hard constraints, such as domain, activity, sequencing, path, and physical constraints, which define the *feasibility* of system behaviour:

$$g_f(\mathbf{F}) := \mathbf{F}(\mathbf{x}, \mathbf{y}, t) - \overline{\mathbf{F}} \leq 0. \tag{2}$$

These hard constraints are typically non-negotiable, as they cannot be changed by the actor (decision maker). Yet, the decision maker can further define constraints on the *acceptability* of solutions through soft ("negotiable") constraints, which are expressed entirely in terms of stakeholder preferences. Any implicit performance limits arise from the actor's choice of reference points used to construct the preference functions, rather than from explicit bounds imposed directly on the performance functions.

For each performance function f_i, the actor can define an individual reference interval

$$[\, f_i^{\text{loc}}\, ;\, f_i^{\text{upc}}\,] \subseteq [\, f_i^{\min}\, ;\, f_i^{\max}\,],$$

where $f_i^{\min}$ and $f_i^{\max}$ denote the global minimum and maximum of f_i over the feasible system space $\mathcal{S}_f$. The endpoints of this interval, $(f_i^{\text{loc}}, f_i^{\text{upc}})$, are associated with preference scores of 100 ("best") and 0 ("worst"), while the mapping of intermediate values is left unconstrained, reflecting the individual actor's preference function over the corresponding performance dimension.

The acceptability of a system solution is expressed component-wise as

$$g_a\big(P_{k,i}(\mathbf{F})\big) := P_{k,i}(\mathbf{F}) - \overline{P}_{k,i} \geq 0, \tag{3}$$

where $\overline{P}_{k,i}$ denotes the minimum acceptable preference (typically 0, though the actor may impose stricter thresholds).
The resulting preference functions over the performance dimensions for actor k are defined as

$$P_{k,i}\big(f_i(\mathbf{x},\mathbf{y},t)\big) \in [0,100], \quad \text{with } f_i(\mathbf{x},\mathbf{y},t) \in [f_i^{\text{loc}}, f_i^{\text{upc}}] \subseteq [\, f_i^{\min}\,;\, f_i^{\max}\,].$$

Finally, the complete design–decision system solution space, encompassing both feasibility and acceptability constraints, is defined as

$$\mathcal{S}_{f,a} := \{\mathbf{x} \mid g_f(\mathbf{x},\mathbf{y},t) \leq 0 \ \wedge \ g_a(P_{k,i}(\mathbf{F})) \geq 0\}\,. \tag{4}$$

NOTES:
(1) By distinguishing *objectively measurable performance functions* (e.g., traffic or noise hindrance, determined by the decision vector $\mathbf{x}$) from *subjectively assessed functions* (e.g., costs or aesthetics, evaluated according to stakeholder perception), system performance is embedded within a design–decision framework that links the decision vector $\mathbf{x}$ to both quantifiable outcomes and human-centered evaluations.
(2) In this formulation, objective functions are not treated as independent optimisation targets. Instead, goal-orientation is fully captured through stakeholder preference functions $P_{k,i}$, which form the basis for decision-making. The performance functions f_i describe what the system *can* (capability), while the preferences express what each actor *wants* (desirability). Decisions therefore arise from the aggregation of preferences rather than from direct optimisation of extrema of f_i. Unlike classical multi-objective optimisation (MOO), which relies on numerical scaling in objective space, ODESYS maps all system behaviour into a common preference space prior to aggregation.
(3) Hard constraints define feasible system behaviour and are non-negotiable, arising from physical laws, technical limits, regulations, or environmental conditions. These are captured by $g_f(\mathbf{x},\mathbf{y},t) \leq 0$, encompassing domain, activity, sequencing, path, and physical constraints, and may implicitly include equality constraints $h_f(\mathbf{x},\mathbf{y},t) = 0$. Soft (negotiable) constraints, in contrast, act on the preference functions $P_{k,i}$ and reflect stakeholder priorities and acceptable performance levels. By defining an acceptability interval $[\, f_i^{\text{loc}}, f_i^{\text{upc}}\,]$, the decision maker implicitly bounds acceptable system performance while maintaining all acceptability reasoning within the preference space.

2. IMAP solver & ODESYS structure in FIVES

In complex socio-technical design problems, multiple stakeholders express their preferences over a set of performance functions $\mathbf{F}(\mathbf{x},\mathbf{y},t)$. Each preference function $P_{k,i}$ captures the desirability of multiple performance capabilities F_i for stakeholder k, potentially weighted by $w'_{k,i}$. To determine an optimal design decision, it is necessary to aggregate all stakeholder preferences into a single aggregated score, while ensuring that all non-negotiable feasibility constraints and negotiable acceptability conditions are respected. Formally, this so-called integrative maximisation of aggregated preferences (IMAP) constraint optimisation statement can be expressed as:

$$\max_{\mathbf{x}} Z = \mathbf{A}\Big(P_{k,i}(\mathbf{F}(\mathbf{x},\mathbf{y},t));\ w'_{k,i}\Big) \tag{5}$$

where $(\mathbf{x},\mathbf{y}) \in S_{f,a}$ is the set of design decisions satisfying all hard feasibility constraints g_f and soft acceptability constraints g_a, and where $\mathbf{A}$ is the **a-fine-aggregator** that aggregates multiple actors preference functions integratively with their performances. Here $\mathbf{A}$ is uniquely defined as a linear weighted centroid operator of the z-normalised $P_{k,i}(\mathbf{x})$ scores: $z_{k,i}(\mathbf{x})$ (see (Wolfert, 2026) for more details on a-fine aggregation). Now we can search for a **best-fit-for-common-purpose** solution, which is the unique design and decision point $\mathbf{x}^*$ that has the highest aggregated weighted normalized preference score:

$$\mathbf{x}^* = \arg\max_{\mathbf{x}} Z(\mathbf{x}), \quad Z(\mathbf{x}) = \sum_{k,i} w'_{k,i}\, z_{k,i}(\mathbf{x}), \quad \sum_{k,i} w'_{k,i} = 1 \tag{6}$$

Solvability is defined as the existence and computability of a design decision vector $\mathbf{x}^*$ that maximises the aggregated desirability Z as an integral function of capability F , subject to hard feasibility constraints and soft acceptability conditions : i.e., Integrative Maximisation of Aggregated Preferences (IMAP). Unlike classical decidability, solvability assumes the availability of a solver and focuses on the resolution of a complex design and decision space. The resulting design-decision engine is the so-called Preferendus, a tool that determines the optimal best-fit design: i.e., that candidate solution with the highest aggregated, integrative, preference point, within the given solution space
To solve the optimisation problem underlying IMAP, the Preferendus employs a fit-for-purpose inter-generational genetic algorithm (GA). This solver is specifically designed to handle normalised preference scores and contextual rank changes inherent to preference aggregation methods such as Tetra. Instead of directly comparing successive generations, the algorithm stores the best-ranked solution of each generation and evaluates these solutions separately in an inter-generational reference set, enabling meaningful comparison of aggregated preferences over time. In addition, the GA allows for user-defined initial solutions and applies a selective re-evaluation step that excludes clearly irrelevant or non-competitive alternatives.Together, these modifications ensure robust convergence toward the design configuration that maximises the integrative aggregated group preference, making the inter-generational GA the core IMAP a-fine-aggregator–based solver engine of the Preferendus tool. This has been implemented in the open-source Preferendus solver (Teuber et al., 2025; van Heukelum et al., 2024), which is accessible via the links provided in the Data Availability section and the Appendix.

Notes:
(1) Unlike classical optimisation tools that explore a Pareto front or multiple non-dominated solutions, ODESYS identifies a *unique design–decision point* $\mathbf{x}^*$, representing the best-fit-for-common-purpose solution within an IMAP structure in FIVES.
(2) ODESYS does not rely on multiple independent min–max objectives that require numerical scaling or monetization, as in classical numerical optimisation. Instead, decision-making is entirely driven by the aggregation of stakeholder preferences, with IMAP solving searching for the integrative solution that maximizes the aggregated preference score.
(3) ODESYS can also be applied to single-objective problems. If a single preference weight $w'_{k,i} = 1$ while all other weights are zero, the IMAP aggregation reduces to a single-objective problem, where the preference directly represents this objective. In this case, the method yields a single optimal solution, analogous to a classical `MINIMIZE` or `MAXIMIZE` operation over a single objective (SOO).
(4) ODESYS' FIVES structure further extends the ODESYS/IMAP 1.0 framework to a broader spectrum of group design-decision applications, explicitly confronting systems complexity and socio-technical interactions. Building on the proven robustness of 'ODESYS/IMAP 1.0', it preserves previously validated outcomes while enabling richer integration of capability, feasibility, desirability, acceptability, and solvability across different multi-actor settings.

Summary — ODESYS structure in FIVES

The ODESYS operator, describing a multifaceted design-decision problem, is defined as

$$\mathrm{OD}\Big(P_{k,i}(\mathbf{F}(\mathbf{x},\mathbf{y},t));\ w'_{k,i}\Big) \tag{7}$$

and is structured in FIVES (framework summary, with reference to all source Equations in Sections 1–2):

$$\begin{array}{ll} \text{(1) Capability (performances)} & \mathbf{F}(\mathbf{x},\mathbf{y},t) \\ \text{(2) Feasibility (hard constraints)} & g_f(\mathbf{x},\mathbf{y},t) \le 0 \\ \text{(3) Desirability (preferences)} & P_{k,i}\big(\mathbf{F}(\mathbf{x},\mathbf{y},t)\big) \\ \text{(4) Acceptability (soft constraints)} & g_a\big(P_{k,i}(\mathbf{F}(\mathbf{x},\mathbf{y},t))\big) \ge \overline{P}_{k,i} \\ \text{(5) Solvability (IMAP)} & \max\limits_{\mathbf{x}} Z = \big(z_{k,i}(\mathbf{x});\ w'_{k,i}\big) \end{array} \tag{8}$$

3. Demonstrative ODESYS/FIVES Examples

In this section, we present two illustrative problems to demonstrate the ODESYS structure in FIVES methodology (see Equations (7)- (8)). The first is an integrative design–decision problem for the installation of a floating wind farm, which has been used as a demonstrator in several previous publications (Teuber et al., 2025; van Heukelum et al., 2023, 2024). The aim is to showcase the further developed and generalised ODESYS/FIVES problem formulation and the associated methodology to solve it, including explicit affine preference aggregation. The objective is therefore to demonstrate the ODESYS/FIVES framework rather than to present detailed application results. For specific design–decision results, the reader is referred to the aforementioned publications or to the ODESYS textbook example in Chapter 8.5 (Wolfert, 2023), as well as to the links provided in the Data Availability section. While the results remain identical, the problem formulation presented here is novel and preserves the previously validated design–decision outcomes.

The second example concerns a fleet allocation decision problem of the Marine Services business unit within the globally operating marine contractor Boskalis, which led to the development of their ODESYS/FIVES-based AlloDyn operational decision-support software. In this case, the methodology is also presented together with a modelling approach for preference-based group decision-making. As with the first example, this paper does not discuss specific decision results for this portfolio optimisation problem, as these are commercially confidential within the context of Boskalis/AlloDyn. Together, these examples provide a concise illustration of the ODESYS/FIVES methodology: the first example offers a detailed, textbook-based demonstration, while the second shows its application in a highly constrained industrial vessel allocation context.

(#1) Integrative design-decision problem - floating wind farm

This demonstrator addresses a complex, socio-technical installation planning problem for a floating wind farm, in which engineering design choices and operational decisions are tightly coupled with the heterogeneous and potentially conflicting interests, namely project duration, cost, fleet utilisation, and emissions. The floating wind farm installation problem, comprising multiple vessels and two concurrent actors (an energy service provider and a marine contractor), is formulated as an open design and decision system (ODESYS) synthesis operator:

$$\mathrm{OD}\Big(P_{k,i}\big(\mathbf{F}(\mathbf{x},\mathbf{y},t)\big);\ w'_{k,i}\Big), \qquad (\mathbf{x},\mathbf{y}) \in \mathcal{S}_{f,a}, \quad k = 1..2,\ i = 1..4. \tag{9}$$

This problem is formulated as an *integrative design–decision problem*, as it simultaneously addresses decision variables related to process management (e.g. vessel selection and deployment) and engineering design choices (e.g. anchor dimensions). We will now define the ODESYS structure in FIVES.

Capability The system capability is represented by a four-dimensional multi-system performance vector

$$\mathbf{F}(\mathbf{x},\mathbf{y},t) = \big[f_1(\mathbf{x},\mathbf{y},t), f_2(\mathbf{x},\mathbf{y},t), f_3(\mathbf{x},\mathbf{y}), f_4(\mathbf{x},\mathbf{y},t)\big],$$

where the performance functions respectively capture project duration, installation cost, fleet utilisation, and vessel-related emissions. Time-dependence is included only where relevant, reflecting that fleet utilisation is evaluated independently of project duration, while the remaining performance functions depend on operational time.

The project is evaluated using four performance functions, denoted f_1 through f_4. The project duration performance function $f_1(x_{1..3}, \mathbf{y}, t)$ determines the total installation time in days as a function of the selected vessel types and quantities, with the overall duration emerging from a discrete event simulation (DES) that captures installation rates, deck capacities, and anchor reloading operations. The installation cost performance function $f_2(x_{1..5}, \mathbf{y}, t_{\text{vessel}}(x_{1..3}))$ calculates the total project cost by combining anchor manufacturing costs (Capex) and time-dependent vessel day rates (Opex), where vessel operating times are obtained from the DES. The fleet utilisation performance function $f_3(x_{1..3}, \mathbf{y})$ evaluates the strategic suitability of the selected vessels by quantifying the combined probability that they could have been more effectively deployed on alternative projects, independent of time. Finally, the vessel emissions performance function $f_4(x_{1..3}, \mathbf{y}, t_{\text{vessel}}(x_{1..3}))$ measures the total project-related CO_2 emissions by summing the time-dependent emissions of all selected vessels, with operational durations obtained from the DES.

Here the controllable decision vector and their domain constraints are defined as:

Table 1: Controllable design vector $\mathbf{x} = (x_1, \ldots, x_5)$ and domain constraints.

$\mathbf{x}$	**Description**	$g_f^{(0)}(x_i)$
x_1	Number small vessels	$0 \leq x_1 \leq 3,\ \ x_1 \in \mathbb{Z}_{\geq 0}$
x_2	Number large vessels	$0 \leq x_2 \leq 2,\ \ x_2 \in \mathbb{Z}_{\geq 0}$
x_3	Number crane barges	$0 \leq x_3 \leq 2,\ \ x_3 \in \mathbb{Z}_{\geq 0}$
x_4	Anchor diameter	$1.5 \leq x_4 \leq 4$
x_5	Anchor penetration length	$2 \leq x_5 \leq 8$

The uncontrollable parameter vector $\mathbf{y}$ comprises the working point force on the anchor, platform type, mooring configuration, anchor type, soil conditions, and mooring line properties, all of which are fixed per scenario or assumed *a priori*.

Feasibility The feasible system solution space is therefore defined as

$$\mathcal{S}_f = \left\{ (\mathbf{x}, \mathbf{y}) \;\middle|\; g_f^{(m)}(\mathbf{x}, \mathbf{y}) \leq 0,\ m = 0..2 \right\}.$$

$g_f^{(1)}$: logical constraint

$$g_f^{(1)}(\mathbf{x}) = -(x_1 + x_2 + x_3) + 1 \leq 0.$$

$g_f^{(2)}$: physical constraint

$$g_f^{(2)}(\mathbf{x}, \mathbf{y}) = F_a(\mathbf{y}) - R_a(x_4, x_5, \mathbf{y}) \leq 0.$$

Desirability There are two actors in the system, each assigned an equal global weight of 0.5. Both actors are, in principle, free to express preferences over all four performance dimensions f_1–f_4; however, in this case, each actor focuses only on the criteria most relevant to their operational objectives. The first actor, the energy service provider, is primarily concerned with f_1 and f_4, with an equal distribution of preferences such that $w_{11} = w_{14} = 0.25$. The second actor, the marine contractor, focuses on f_2 and f_3, also with equal distribution, i.e., $w_{22} = w_{23} = 0.25$. See Figure 1 for a schematic illustration of the preference curves for both actors. NOTES: The preference structure adopted in this example is intentionally simple and symmetric with respect to the four performance functions. In general, the ODESYS methodology enables asymmetric preference formulations and heterogeneous stakeholder interests, allowing actors to express conflicting priorities and differentiated weight structures within a fully participative decision-making framework that formally integrates global and local weights (see (Teuber & Wolfert, 2024) for methodological details and conjoint-analysis-based preference elicitation).

Acceptability For acceptability, no explicit lower or upper thresholds are imposed on the performance functions (e.g., no cost cap for f_2). Consequently, for each performance dimension i, the preference function spans the full range defined by the observed minimum and maximum system performance values:

$$P_{k,i}\big(f_i(\mathbf{x}, \mathbf{y}, t)\big) \in [0, 100], \qquad f_i(\mathbf{x}, \mathbf{y}, t) \in [f_i^{\min},\ f_i^{\max}].$$

In the absence of explicit acceptability constraints ($g_a \geq 0$), all feasible performance values are admissible, and their associated normalized preferences span the full interval $[0, 100]$. Here, 0 represents the least preferred alternative relative to the others, while 100 represents the most preferred. Importantly, a value of 0 denotes the lowest relative preference score and does not imply the absence of performance on the corresponding criterion.

This formulation ensures that all feasible system performances are fully represented in the preference space. If explicit acceptability constraints are introduced—such as a maximum project duration, budget ceiling, or emission limit—the admissible range of the corresponding performance functions is reduced accordingly, thereby restricting the acceptable (but not necessarily feasible) decision space.

Solvability The resulting design–decision solution space is therefore defined as

$$\mathcal{S}_{f,a} := \left\{ \mathbf{x} \;\middle|\; g_f^{(m)}(\mathbf{x}, \mathbf{y}, t) \leq 0 \;\wedge\; g_a^{(n)}(P_{k,i}(\mathbf{F})) \geq 0, \quad m = 0..2,\; n = 1..8 \right\}.$$

In this case, $\mathcal{S}_{f,a}$ is bounded solely by the system feasibility constraints g_f, while the acceptability function g_a spans the full preference range between 0 and 100. The ODESYS structure-in-FIVES problem formulation can be schematically illustrated in Fig. 1, which visualizes how system performance, actor preferences, and constraints jointly define the solution space.

The final objective is to identify the unique design–decision vector $\mathbf{x} \in \mathcal{S}_{f,a}$ that maximizes the aggregated preference score. This can be achieved using the affine aggregation operator as part of Eqs. (5) and (6). By applying an appropriate **search algorithm**, e.g., an intergenerational genetic algorithm (van Heukelum et al., 2024), the optimal design–decision vector $\mathbf{x}^*$ can be determined.

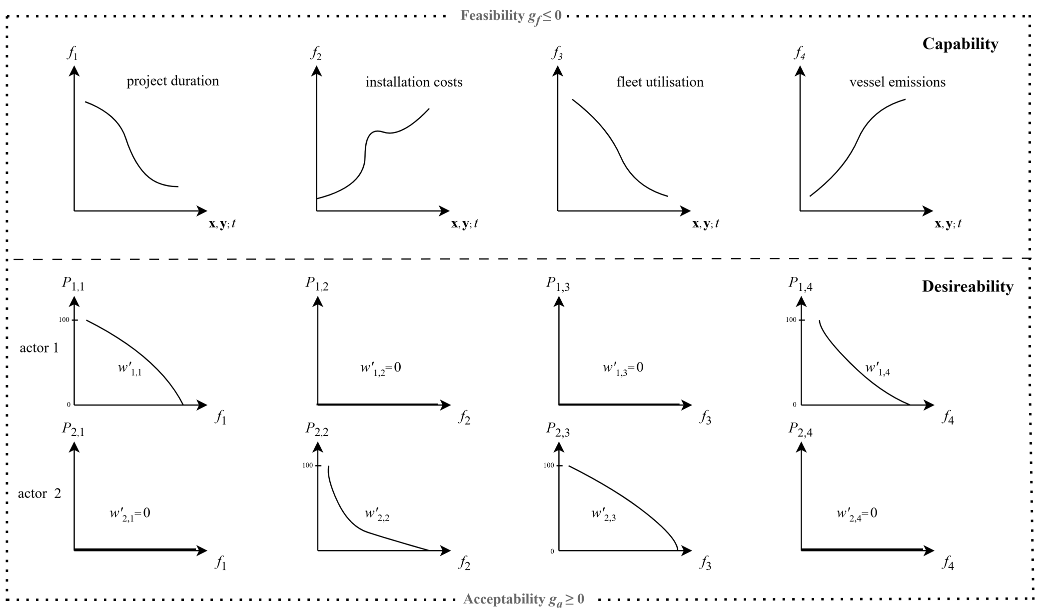

Figure 1: ODESYS problem schematization

(#2) Multi-constrained decision problem – dynamic vessel allocation
Within Boskalis, the ODESYS methodology has been applied to a multi-constrained dynamic vessel allocation problem, resulting in the development of the *AlloDyn* decision-support tool. AlloDyn is embedded within the Vessel Operations Management System (VOMS) of Boskalis' Marine Services business unit managing a global fleet of approximately 30 vessels. The system allocates a portfolio of new commercial tasks or projects, obtained by the commercial department, to the available vessels worldwide, balancing multiple criteria such as voyage distance, make-span, vessel capabilities, maintenance, mobilisation costs, and operational priorities.
The focus of this illustrative problem is not on the operational outcomes produced by AlloDyn, but on formally describing the underlying multifaceted decision problem and its multiply constrained solution space. The emphasis is therefore on managerial decision-making related to vessel allocation, rather than on the design or modification of physical assets. For simplicity, the problem is considered with two 'çonflicting' actors ($k = 1..2$) representing the operational and commercial decision-making teams of the business unit, with four performance interests ($i = 1..4$), and is formulated as an ODESYS synthesis operator:

$$\mathrm{OD}\Big(P_{k,i}\big(\mathbf{F}(\mathbf{x},\mathbf{y})\big);\ w'_{k,i}\Big), \qquad (\mathbf{x},\mathbf{y}) \in \mathcal{S}_{f,a}, \quad k = 1..2, \quad i = 1..4. \tag{10}$$

This formulation represents an integrative vessel allocation decision problem, simultaneously addressing decision variables related to vessel management (e.g., selection and scheduling) within an operational fleet context under multiple constraints. The two actors have a conflict of interest within the overall bottom line of the business unit: the operations unit aims to minimise operational costs, whereas the commercial team seeks to maximise the number of projects within a portfolio over a given time span, thereby optimising the make-span. The system performance vector $\mathbf{F}(\mathbf{x},\mathbf{y})$ captures the relevant system performances, while the aggregated actor preferences $\mathbf{A}\big(P_{1..2;1..4}(\mathbf{F}); w'_{1..2;1..4}\big)$, expressed through the ODESYS framework, determine the final allocation decisions. The full problem formulation, including all variables, parameters, and constraints, is provided in the Appendix; here, only the main ODESYS structures in FIVES are outlined.
NOTE: This formulation represents a highly constrained combinatorial decision problem, characterised by tightly coupled temporal, spatial, and resource constraints. In practice, such problems are typically solved using constraint programming (CP) techniques. In this work, however, they are embedded within the ODESYS framework to enable pure preference-based group decision-making, rather than hierarchical or purely numerical multi-objective optimisation across multiple actors and performance dimensions. In other words, conventional CP or MIP approaches are limited to numerical constructs and optimisation frameworks and consequently miss the opportunity to treat the problem as a decision game, in which actors' preferences and interests are fully integrated and pivotal to this game.
We now provide a summary of the ODESYS structure in FIVES .

Capability The system capability is described by a four-dimensional performance vector

$$\mathbf{F}(\mathbf{x},\mathbf{y}) = \big[f_1(\mathbf{x},\mathbf{y}), f_2(\mathbf{x},\mathbf{y}), f_3(\mathbf{x},\mathbf{y}), f_4(\mathbf{x},\mathbf{y})\big],$$

representing total mobilisation distance $f_1(\mathbf{x},\mathbf{y})$, total cost $f_2(\mathbf{x},\mathbf{y})$ (i.e., the sum of all mobilization and standby costs, excluding the one-to-one activity sailing costs, to complete the activity portfolio), total fuel consumption $f_3(\mathbf{x},\mathbf{y})$, and total make-span $f_4(\mathbf{x},\mathbf{y})$ (i.e., the total time required to complete the entire portfolio of activities). The controllable design–decision vector is $\mathbf{x} = (x_1..x_6)$, while the uncontrollable parameter vector is $\mathbf{y} = (y_1..y_{15})$. The full analytical expressions for f_1–f_4, including their dependence on $\mathbf{x}$ and $\mathbf{y}$, as well as the domain constraints $g_f^{(0)}(\mathbf{x}, \mathbf{y})$, are provided in the Appendix.

Feasibility For the multi-constrained vessel allocation problem, system feasibility is enforced through a set of constraints that ensure: - domain validity (variable bounds on the decision vector $\mathbf{x}$ and parameter vector $\mathbf{y}$), - assignment feasibility (correct allocation of vessels to activities), - sequencing and temporal consistency (activities follow valid temporal orders), - and path continuity (feasible travel paths between activity locations).
The feasible system solution space is therefore defined as

$$\mathcal{S}_f = \left\{ (\mathbf{x}, \mathbf{y}) \;\middle|\; g_f^{(m')}(\mathbf{x}, \mathbf{y}) \le 0,\ m' = 0..13 \right\}.$$

Here, $g_f^{(0)}$ represents the domain constraints for the decision vector $\mathbf{x}$ and the parameter vector $\mathbf{y}$, while constraints $g_f^{(1)}$–$g_f^{(13)}$ capture all remaining activity, sequencing, and path constraints. All individual constraints are explicitly specified in the Appendix.

Desirability In the present demonstrator, there are two actors in the system, each assigned an equal global weight of 0.5. The first actor, representing the operations team, is solely interested in the cost performance function f_2, with weight $w_{12} = 0.5$, while all other weights for this actor are zero. The second actor, representing the commercial team, is solely interested in the make-span performance function f_4, with weight $w_{24} = 0.5$, and all other weights for this actor are zero. System desirability is evaluated using actor-specific preference functions over each performance dimension, which are linear for this case[1]. The linear preference curves reflect the relative desirability of performance outcomes for each actor, without yet imposing acceptability limits.
NOTE: The preference structure in this demonstrator is intentionally simplified to highlight actor-specific interests using linear curves. In general, ODESYS allows asymmetric, free-form preference functions and heterogeneous stakeholder interests, enabling actors to express conflicting priorities and differentiated weight structures within a fully participative decision-making framework that formally integrates global and local weights (see (Teuber & Wolfert, 2024; Teuber et al., 2025)).

Acceptability Acceptability is imposed directly on the normalized preference values rather than on the raw performance functions. For this demonstrator, each performance dimension is mapped linearly to a preference scale from 0 to 100:

$$P_{k,i}\big(f_i(\mathbf{x}, \mathbf{y})\big) = 100 \cdot \frac{f_i(\mathbf{x}, \mathbf{y}) - f_i^{\min}}{f_i^{\max} - f_i^{\min}},$$

where the endpoints correspond to the minimum and maximum feasible performance over the solution space:

$$f_i^{\min} = \min_{\mathbf{x} \in \mathcal{S}_{f,a}, \mathbf{y}} f_i(\mathbf{x}, \mathbf{y}), \qquad f_i^{\max} = \max_{\mathbf{x} \in \mathcal{S}_{f,a}, \mathbf{y}} f_i(\mathbf{x}, \mathbf{y}).$$

[1] Within Boskalis, where this ODESYS system operates as part of the *AlloDyn* software, one uses different business-specific and customized preference functions including business-specific thresholds.

Here, 0 represents the least preferred alternative and 100 the most preferred; a score of 0 denotes the lowest relative preference and does not imply zero performance. In this demonstrator, no explicit minimum acceptable preference levels are imposed ($\overline{P}_{k,i} = 0$), so the acceptability constraints do not further restrict the feasible space, and the combined feasible–acceptable solution space reduces to $\mathcal{S}_{f,a} = \mathcal{S}_f$. When minimum preference thresholds are specified, only solutions satisfying $g_a^{(n')}(P_{k,i}(\mathbf{F})) \geq \overline{P}_{k,i}$ are retained. These constraints restrict the acceptable solution space but do not affect system feasibility.

Solvability The resulting multi-constrained design–decision solution space is

$$\mathcal{S}_{f,a} := \left\{ \mathbf{x} \;\middle|\; g_f^{(m')}(\mathbf{x}, \mathbf{y}) \leq 0 \;\wedge\; g_a^{(n')}\big(P_{k,i}(\mathbf{F})\big) \geq 0,\; m' = 0..13,\; n' = 1..8 \right\}.$$

The final objective is to identify the unique design–decision vector $\mathbf{x}^* \in \mathcal{S}_{f,a}$ that maximizes the aggregated preference score, using the affine aggregation operator as part of Eqs. (5) and (6). This is achieved using the extended IMAP/Preferendus ((see Data Availability section and (van Heukelum et al., 2024)), which here incorporates an intergenerational Biased Random-Key Genetic Algorithm (BRKGA) with a pseudo-random decoder. Candidate vectors in $[0, 1)^n$ are mapped to complete schedules while explicitly enforcing all system constraints (e.g., assignment uniqueness, temporal precedence) during evolution. Biased inheritance accelerates convergence by preferentially propagating elite solutions. This extended GA, as part of the Preferendus, explores the feasible–acceptable solution space $\mathcal{S}_{f,a}$ and evaluates candidates through the a-fine aggregation of z-normalized preference scores, ultimately yielding the best-fit-for-common-purpose design–decision vector $\mathbf{x}^*$, i.e., the maximum aggregated-preference decision vector solution.

4. Conclusions

The demonstrative examples above illustrate that conventional multi-objective approaches, when applied without integrated preference modelling, cannot guarantee a unique, consistent, and mathematically valid group decision outcome. In contrast, the ODESYS structure in FIVES implements a pure performance–preference paradigm, where (1) capability (system performance), (2) feasibility (hard constraints), (3) desirability (stakeholder preferences) and (4) acceptability (negotiable constraints), are fully integrated within a single associative decision space, while (5) solvability is achieved by integratively maximising the aggregated preference (IMAP). For this, preferences are mapped into a single preference domain and aggregated via a uniquely defined, PFM-consistent linear aggregation operator, ensuring that all relative differences are preserved and that the resulting group-optimal solution simultaneously satisfies the four MOO–MCDM conditions—Preference-Key, Integration, Association, and Uniqueness—yielding a single, consistent, and fully group decision-valid outcome. By contrast, all other methods reviewed in the references cited in this paper—including CP-, MIP-, and conventional MOO approaches—do not satisfy the four MOO–MCDM conditions and therefore cannot guarantee a decision-valid solution, as also independently noted in (Ferdous et al., 2024; Pajasmaa et al., 2025). ODESYS overcomes these limitations through its integrated preference-based framework. Moreover and importantly, the framework preserves individual design freedom within the feasible and acceptable solution space, enabling actors to express their priorities while contributing to a collective best-fit, resulting in a synthesis of common purpose across the system's 'infinite' physical possibilities.

Through its structure-in-FIVES formulation, ODESYS operationalises this methodology for complex, multi-system design and decision problems, rigorously enforcing both hard feasibility constraints and soft acceptability conditions. IMAP identifies the unique best-fit-for-common-purpose design–decision vector using a intergenerational genetic search algorithm (GA), accommodating multiple actors with diverse preferences. In doing so, it finds the outcome with maximum aggregated preference, maintains full traceability and participatory engagement and enables forward-looking design decision-making rather than selecting from a curated numeric set, which unveils the illusion of free choice in conventional MCDM. Taken together, these results establish the IMAP solver and ODESYS/FIVES framework as a rigorous, preference-consistent, and operationally validated methodology for multi-objective, multi-actor design–decision-making. It demonstrates how system performance *capability*, stakeholder preference *desirability*, and *feasibility* versus *acceptability* can be coherently integrated to produce decisions that are simultaneously transparent, associative, and *solvable* as a best-fit-for-common-purpose. Consequently, when individual actors are willing to give in on their pure self-interests, the group outcome achieves a higher aggregated preference, transforming creative conflicts into a shared 'yes' — without deceiving the realities of the physical system. This sets the stage for the final conclusions, highlighting both the theoretical and practical contributions of the ODESYS to confronting complex, socio-technical system developments. In doing so, ODESYS turns MOO-MCDM upside down and right side up, leveraging the synergy of 'systems-thinking-social' and 'design-thinking-slow' to deliver a best-fit-for-and-from-the-whole within reach: a conscience of freedom!

Disclosure Statement

The author declares that there are no competing interests.

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Data Availability

The Preferendus software tool, including one of the ODESYS demonstrators (a floating wind farm) discussed in this paper, is publicly available via (Wolfert, 2023) (see DA4 in Chapter 8.5) and its GitHub repository `https://github.com/TUDelft-Odesys`. Moreover, an extended and stochastic variant of this design–decision example, published in (Teuber et al., 2025), is accessible—in accordance with confidentiality agreements—via the repository `https://github.com/Boskalis-python/ODYCON`. It uses the latest Preferendus version and employs the A-Fine-Aggregator for preference-based optimisation; the algorithm is available at: `https://github.com/Boskalis-python/a-fine-aggregator`.

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Appendix

Let the (vessel) allocation decision problem be defined by the ODESYS system structure in FIVES as:

$$\mathrm{OD}\Big(P_{k,i}\big(\mathbf{F}(\mathbf{x},\mathbf{y})\big);\ w'_{k,i}\Big), \qquad (\mathbf{x},\mathbf{y}) \in \mathcal{S}_{f,a} \tag{A1}$$

Here, $\mathrm{OD}(\cdot)$ is the ODESYS synthesis operator, $\mathcal{S}_{f,a}$ represents the feasible and acceptable system solution space, and $\mathbf{F}(\mathbf{x},\mathbf{y})$ is the system performance (capability) vector. The functions $P_{k,i}(\cdot)$ define actor specific preference functions over performance dimension i for actor k, while $w'_{k,i}$ denote the associated local preference weights.

(1) Capability

For the vessel allocation problem, multi-system performance is captured by the following capability functions:

$$\mathbf{F}(\mathbf{x},\mathbf{y}) = \big[f_1(\mathbf{x},\mathbf{y}), f_2(\mathbf{x},\mathbf{y}), f_3(\mathbf{x},\mathbf{y}), f_4(\mathbf{x},\mathbf{y})\big]. \tag{A2}$$

Mobilization distance is defined as:

$$f_1(\mathbf{x},\mathbf{y}) = \sum_{r\in R}\sum_{r'\in R} x_4[r,r'] \cdot y_{10}[\ell^{\text{end}}(r), \ell^{\text{start}}(r')]$$

while the total cost (i.e., the sum of all mobilization and standby costs, excluding the one-to-one activity sailing costs, to complete the activity portfolio) is given by:

$$f_2(\mathbf{x},\mathbf{y}) = \sum_{r\in R}\sum_{r'\in R} x_4[r,r'] \cdot \gamma\left(x_3[r], \ell^{\text{end}}(r), \ell^{\text{start}}(r'), x_6[r,r'], \delta_{r,r'}\right).$$

Fuel consumption is modelled as:

$$f_3(\mathbf{x},\mathbf{y}) = \sum_{r\in R}\sum_{r'\in R} x_4[r,r'] \cdot y_{12}[x_3[r]](x_6[r,r']) \cdot \theta(x_3[r], \ell^{\text{end}}(r), \ell^{\text{start}}(r'), x_6[r,r'])$$

and the total make-span (i.e., the total time required to complete the entire portfolio of activities) is expressed as:

$$f_4(\mathbf{x},\mathbf{y}) = \max_{a\in A}(x_1[a] + y_1[a]) - \min_{a\in A}(x_1[a])$$

Here, the controllable decision vector is defined as $\mathbf{x} = (x_1, \ldots, x_6)$, with components corresponding to timing, location, vessel assignment, sequencing, and sequence initiation decisions; the associated domain constraints are summarised in Table 2. The uncontrollable parameter vector is defined as $\mathbf{y} = (y_1, \ldots, y_{15})$, where each parameter captures exogenous system characteristics such as activity durations, time windows, spatial information, precedence relations, vessel capabilities, and sailing characteristics, and whose associated domains and feasibility constraints are listed in Table 3. Finally, the index sets used in the formulation are defined in Table 4, representing the vessels, activities, roles, and geographical locations relevant to the problem; the activity set is further partitioned by type, yielding a more compact role-based formulation than a binary vessel–activity assignment matrix.

Table 2: Domain constraints for the simplified decision vector $x = (x_1, \ldots, x_6)$

x	**Description**	$g_f^{(0)}(x_i)$
x_1	Start time of activity $a \in A$	$\underline{T}_a \leq x_1[a] \leq \overline{T}_a$
x_2	Location choice for maintenance activity $a \in A_{maint}$	$x_2[a] \in L_a$
x_3	Vessel assigned to role $r \in R$	$x_3[r] \in \mathcal{D}_r$
x_4	Sequencing variable: role r' follows r	$x_4[r, r'] \in \{0, 1\}$
x_5	Sequence start indicator: role r is first in sequence	$x_5[r] \in \{0, 1\}$
x_6	Average speed for sub-route (r, r')	$x_6[r, r'] \in [\underline{s}, \overline{s}]$

Table 3: Parameter vector $\mathbf{y} = (y_1, \ldots, y_{15})$ and their domain constraints

y	**Description**	$g_f^{(0)}(y_i)$
y_1	Duration of activity a	$y_1[a] \geq 0$ (Days)
y_2	Start time window for activity a	$y_2[a] = [\underline{y_2[a]}, \overline{y_2[a]}] \geq 0$
y_3	Start location for towing activity $a \in A_{tow}$	$y_3[a] \in$ Locations
y_4	End location for towing activity $a \in A_{tow}$	$y_4[a] \in$ Locations
y_5	Locations for maintenance activity $a \in A_{maint}$	$y_5[a] \subseteq$ Locations
y_6	Predecessor of activity a	$y_6[a] \in A \cup \{\emptyset\}$
y_7	Parent activity of role r	$y_7[r] \in A$
y_8	Vessel domain for role r	$y_8[r] \subseteq V$
y_9	Set of roles belonging to activity a	$y_9[a] = \{r \in R \mid \alpha(r) = a\}$
y_{10}	Sailing distance from location ℓ to ℓ'	$y_{10}[\ell, \ell'] \geq 0$
y_{11}	Daily mobilisation rate for vessel v	$y_{11}[v] \geq 0$
y_{12}	Fuel consumption rate function for vessel v	$y_{12}[v](s) \geq 0,\ \forall s \in [y_{13}[v]^{\min}, y_{13}[v]^{\max}]$
y_{13}	Feasible sailing speed range for vessel v	$y_{13}[v] = (s_v^{(0)}, s_v^{(1)}, \ldots, s_v^{(\lvert S_v \rvert - 1)}),\ s_v^{(k)} > 0$
y_{14}	Fuel price for vessel v	$y_{14}[v] \geq 0$
y_{15}	Standby cost discount factor for vessel v	$y_{15}[v] \in [0, 1]$

Table 4: All sets in the fleet allocation problem.

Set	**Description**
$V = \{v_0, v_1, \ldots, v_n\}$	Set of n vessels
$A = \{a_0, a_1, \ldots, a_m\}$	Set of m activities
$A^{\text{tow}} \subseteq A$	Subset of towing activities
$A^{\text{maint}} \subseteq A$	Subset of maintenance activities
$R = \{r_0, r_1, \ldots, r_p\}$	Set of p roles
$L = \{\ell_0, \ell_1, \ldots, \ell_q\}$	Set of q locations

Notes:

- $\theta(v, \ell, \ell', s)$ is the sailing duration for vessel v to travel from location ℓ to ℓ' at average speed s, defined by
$$\theta(v, \ell, \ell', s) = \left\lceil \frac{y_{10}[\ell, \ell']}{24 \cdot s} \right\rceil$$

- $\delta_{r,r'} = x_1[y_7[r']] - (x_1[y_7[r]] + y_1[y_7[r]])$ denotes the total transition time between role r and r', where $y_7[r]$ denotes the parent activity of role r.

- $\gamma(v, r, r', s, \delta_{r,r'})$ is the cost for vessel v to travel from role r to r' at speed s with a total travel duration of $\delta_{r,r'}$, defined by
$$\begin{aligned}\gamma(v, r, r', s, \delta_{r,r'}) = \theta(v, \ell^{\text{end}}(r), \ell^{\text{start}}(r'), s) \cdot \Big(y_{11}[v] + y_{14}[v] \cdot y_{12}[v](s) - y_{15}[v] \cdot y_{11}[v]\Big) \\ + y_{15}[v] \cdot y_{11}[v] \cdot \delta_{r,r'}\end{aligned}$$

- $\underline{s} = \min_{v \in V}(y_{13}[v])$, $\overline{s} = \max_{v \in V}(y_{13}[v])$ are the min and max available speeds across all vessels.

- The start and end locations of role r are defined as
$$\ell^{\text{start}}(r) = \begin{cases} y_3[y_7[r]] & \text{if } y_7[r] \in A^{\text{tow}} \\ x_2[y_7[r]] & \text{if } y_7[r] \in A^{\text{maint}} \end{cases}$$
$$\ell^{\text{end}}(r) = \begin{cases} y_4[y_7[r]] & \text{if } y_7[r] \in A^{\text{tow}} \\ x_2[y_7[r]] & \text{if } y_7[r] \in A^{\text{maint}} \end{cases}$$
so that role locations are determined either by fixed parameters (y_3, y_4) for towing activities or by the decision variable x_2 for maintenance activities.

(2) Feasibility

For the multi-constrained vessel allocation problem, the feasibility constraints, indexed by m' (activity, sequencing, and path), are written as a feasibility function $g_f^{(m')}(\mathbf{x}, \mathbf{y}) \leq 0$. Then the feasible system solution space is defined as:

$$\mathcal{S}_f = \left\{ (\mathbf{x}, \mathbf{y}) \;\middle|\; g_f^{(m')}(\mathbf{x}, \mathbf{y}) \leq 0,\ m' = 0..13 \right\}. \tag{A3}$$

Within $\mathcal{S}_f$, the following constraints are identified:

Activity Constraints No vessel may be assigned to more than one role within the same activity:
$$g_f^{(1)}(\mathbf{x}, \mathbf{y}) = x_3[r] - x_3[r'] \neq 0, \quad \forall r, r' \in R,\ r \neq r'.$$
An activity cannot start before its predecessor has finished:
$$g_f^{(2)}(\mathbf{x}, \mathbf{y}) = x_1[a] - \big(x_1[y_6[a]] + y_1[y_6[a]]\big) \geq 0, \quad \forall a \in A:\ y_6[a] \neq \emptyset.$$

Sequencing Constraints In the sequence of activities there can be at most one successor:

$$g_f^{(3)}(\mathbf{x}, \mathbf{y}) = \sum_{r' \in R} x_4[r, r'] - 1 \leq 0, \quad \forall r \in R,$$

and at most one predecessor

$$g_f^{(4)}(\mathbf{x}, \mathbf{y}) = \sum_{r \in R} x_4[r, r'] - 1 \leq 0, \quad \forall r' \in R.$$

There can not be self-loops:

$$g_f^{(5)}(\mathbf{x}, \mathbf{y}) = x_4[r, r] \leq 0, \quad \forall r \in R.$$

If role r' follows r, they must be assigned to the same vessel:

$$g_f^{(6)} = x_4[r, r'] = 1 \to x_3[r] = x_3[r']$$

Transitions between roles of the same activity are prevented, as such roles are performed simultaneously rather than sequentially:

$$g_f^{(7)}(\mathbf{x}, \mathbf{y}) = x_4[r, r'] \leq 0, \quad \forall r, r' \in R : \; y_7[r] = y_7[r'].$$

There can not be temporal precedence in sequences

$$g_f^{(8)}(\mathbf{x}, \mathbf{y}) = x_1[y_7[r]] - x_1[y_7[r']] < 0, \quad \forall r, r' \in R : \; x_4[r, r'] = 1.$$

There should be sufficient travel-time for consecutive roles:

$$g_f^{(9)}(\mathbf{x}, \mathbf{y}) = x_1[y_7[r']] - \Big(x_1[y_7[r]] + y_1[y_7[r]] + \theta(x_3[r], \ell^{\text{end}}(r), \ell^{\text{start}}(r'), y_{13}[x_3[r]]^{max})\Big) \leq 0,$$

where $y_{13}[x_3[r]]^{max}$ is the maximum feasible speed for the selected vessel, and for all $r, r' \in R$ such that $x_4[r, r'] = 1$ and $x_3[r] = x_3[r']$.

Path Constraints Each vessel must have exactly one sequence start:

$$g_f^{(10)}(\mathbf{x}, \mathbf{y}) = \sum_{r \in R} [x_3[r] = v] \left[\sum_{r' \in R} x_4[r', r] = 0\right] = 1, \quad \forall v \in V$$

and exactly one sequence end:

$$g_f^{(11)}(\mathbf{x}, \mathbf{y}) = \sum_{r \in R} [x_3[r] = v] \left[\sum_{r' \in R} x_4[r, r'] = 0\right] = 1, \quad \forall v \in V.$$

For each vessel, the assigned roles must together form exactly one continuous path (path cardinality condition):

$$g_f^{(12)}(\mathbf{x}, \mathbf{y}) = \sum_{r \in R} \sum_{r' \in R} x_4[r, r'] \cdot [x_3[r] = v] - \max\left(0, \; \sum_{r \in R} [x_3[r] = v] - 1\right) = 0, \quad \forall v \in V.$$

If role r' follows r for vessel v, the chosen speed on that sub-route must be in the set of available sailing speeds of that vessel.

$$g_f^{(13)}(\mathbf{x}, \mathbf{y}) = (x_4[r, r'] = 1 \;\wedge\; x_3[r] = v) \;\Rightarrow\; x_6[r, r'] \in y_{13}[v] \quad \forall v \in V, \; \forall r, r' \in R$$